Markowitz Versus the Talmudic Portfolio Diversification Strategies

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The diversification theory advocated by Markowitz [1952] asserts that the optimal diversification strategy is a function of the means, variances, and pair-wise correlations of risky assets. About 1,500 years ago, contrary to Markowitz's theory, the Babylonian Talmud made the following diversification recommendation:[1] "A man should always place his money, one third in land, a third into merchandise, and keep a third in hand."[2] We call this naïve diversification strategy the 1/3 rule and, when n assets are available, the $1/n$ rule. Even though no question remains that Markowitz is theoretically correct, it is still interesting to empirically test the Talmudic advice. The following two factors may provide an edge to each of these competing diversification strategies:

1) Number of assets included in the portfolio
2) Stability of the assets' parameters over time

Employing the $1/n$ rule for investment purposes has the disadvantage of not utilizing information on the various parameters, but by the same token, it has the advantage of not being biased in relying on historical parameters that may be much different from future relevant parameters. This is especially true because the mean-variance (M-V) diversification strategy is very sensitive to possible sampling errors. Indeed, Best and Grauer [1991] concluded that "a surprisingly small increase in the mean of just one asset drives half of the securities from the portfolio."

Is it better to employ the $1/n$ rule and avoid choosing the wrong portfolio mix due to the sampling errors of the various parameters, or is it better to employ the M-V rule, which takes into account the various parameters, albeit with some possible sampling errors? This is the empirical question we address in this article. With in-sample analysis, the M-V strategy, by definition, is superior. However, with the more relevant out-of-sample analysis, we show that the Talmudic wisdom is correct for relatively small portfolios and that the Markowitz theory prevails for relatively large portfolios.

METHODOLOGY

In the following analysis, we confined ourselves to positive portfolios (i.e., short sales were not allowed). This framework is suitable for institutional investors as well as for many individual investors. The analysis can easily be extended, however, to incorporate short sales. The M-V efficient portfolios were found as follows:[3]

$$\text{Min } \underline{X}^T V \underline{X}$$

s.t.

$$\begin{aligned} &\text{a) } \underline{X} \geq 0 \\ &\text{b) } \underline{X}^T \underline{\overline{R}} + (1 - \underline{X}^T \underline{1})r = \overline{R}_p \end{aligned} \tag{1}$$

where $\underline{X}$ is the vector investment proportions, V is the sample variance-covariance matrix, $\overline{R}_i$ is the sample mean rate of return on the ith asset, $\overline{R}_p$ is the portfolio mean rate of return, and r is the riskless interest rate. For the $1/n$ portfolio, we have

$$\begin{aligned} &\text{Variance} = \underline{P}^T V \underline{P} \\ &\text{Mean} = \underline{P}^T \underline{R} + (1 - \underline{P}^T \underline{1})r = \overline{R}_p \\ &\text{where } \underline{P}^T = (1/n, 1/n \ldots 1/n) \end{aligned} \tag{2}$$

and, of course, no optimization is involved in this case.

We studied the period from 1996 to 2007. For the in-sample analysis, we solved Equations (1) and (2) for the whole period. We first derived the M-V efficient set as defined in Equation (1). Next, we calculated the mean and variance of the $1/n$ rule portfolio. The loss induced by the employment of the $1/n$ rule is measured by the vertical distance between the $1/n$ rule portfolio and the M-V efficient frontier. The vertical distance is the mean return lost due to the employment of the $1/n$ rule for a given risk.

We then conducted the more relevant out-of-sample analysis. We divided the whole period T (1991–2007) in the middle, so that the period up to T_1 is used to estimate the various parameters with which we constructed the efficient M-V portfolios, and period $T_1 - T$ is the out-of-sample period for which we compared the performance of the M-V strategy and the $1/n$ rule. We calculated the out-of-sample period ($T_1 - T$) standard deviation of the $1/n$ rule portfolio. For the same standard deviation, we then measured the loss/gain of the $1/n$ rule by the vertical distance between the M-V and $1/n$ rule portfolios. We report, for comparison only, the performance of the ex ante portfolio, which is a portfolio based on the second-period parameters. Of course, this portfolio is unattainable because the ex ante parameters are unknown. As a result, we have three portfolios: the in-sample, out-of-sample, and ex ante unattainable portfolio. The in-sample Markowitz portfolio dominates the naïve portfolio, by definition. However, out of sample it is not obvious which portfolio dominates.

DATA AND RESULTS

Our data are monthly value-weighted returns on the 30 Fama–French industry portfolios.[4] For a given risk, the in-sample results reveal that the M-V portfolio outperforms, as expected, the $1/n$ rule by 0.062% to 0.444% a month, depending on the number of assets, n, in the portfolio (for more details, see Levy and Duchin [2003]).[5]

For practical purposes, however, the in-sample comparison is irrelevant. While investors may employ historical sample estimates of the various parameters, they may actually realize future returns that are determined by a different set of parameters, because of the parameters' instability over time. In terms of relevancy for investors, the Talmudic wise people have a point, as we will discuss soon.

In the out-of-sample analysis, the monthly returns of the first period were employed to establish the M-V–efficient diversification strategy. We then used these diversification weights to calculate the out-of-sample portfolio's mean return and variance. Next, we calculated the mean and variance of the naïve portfolio and compared its performance with that of the Markowitz portfolio.

In order to compare the performance of the M-V and $1/n$ rule portfolios, we calculated the standard deviation of the $1/n$ rule portfolio, as shown in Column 1 of Exhibit 1. Then, using an M-V portfolio with the same standard deviation as the $1/n$ portfolio, we compared their mean returns. Column 2 of Exhibit 1 gives the average return of the optimal ex ante M-V portfolio using the second-period parameters with the same risk of the $1/n$ rule portfolio. As previously explained, this portfolio is not attainable because it is based on the ex ante unknown parameter; it is given only for comparison with the other two relevant and attainable portfolios. Columns 3 and 4 of Exhibit 1 provide the average of the two competing relevant portfolios, and Column 5 gives the loss or gain.

As it turns out, the Talmudic wise people knew what they were talking about. In some cases, the naïve portfolio provides a higher out-of-sample average return than the M-V portfolio. This is particularly true for individual investors who hold a relatively small number of

Exhibit 1

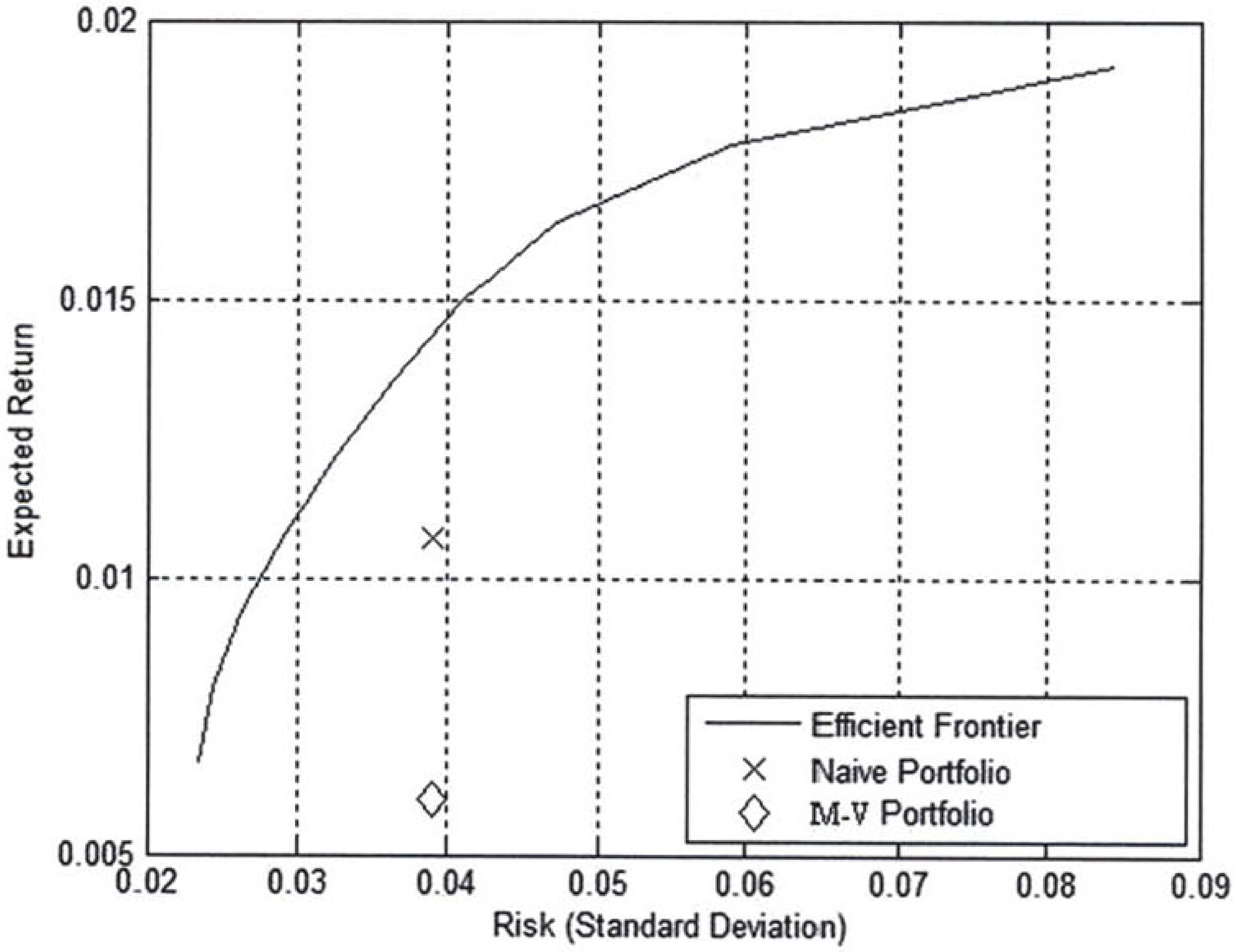
Out-of-Sample Gain/Loss under the Mean-Variance Approach

Number of Assets	Standard Deviation of the Naïve Portfolio (%)	Optimal Ex Ante Average Return (%)*	Ex Ante M-V Average Return (%)	Ex Ante Naïve Average Return (%)**	Difference in Average Returns: Naïve – M-V
	(1)	(2)	(3)	(4)	(5) = (4) – (3)
$n = 15$	3.912	1.442	0.599	1.071	0.472
$n = 20$	3.998	1.679	0.999	1.255	0.256
$n = 25$	4.060	1.700	1.010	1.146	0.136
$n = 30$	3.903	1.646	1.092	1.080	–0.012

*Notes: This exhibit reports the out-of-sample loss induced by following the naïve investment rule (1/n rule) rather than the optimal mean-variance investment rule. *This ex ante portfolio is derived with the second-period parameters and hence is unavailable. It is given for comparison only to the other two portfolios, which are available. **The standard deviation of this portfolio equals the standard deviation of the corresponding naïve portfolio.*

Exhibit 2

Out-of-Sample Mean-Variance Analysis of the 1/*n* rule, *n* = 15



Note: This exhibit compares the out-of-sample performance of the 1/n rule and the mean-variance efficient portfolio, relative to the "true" out-of-sample mean-variance efficient frontier (which is unavailable), when the number of available assets is 15. Data consist of monthly returns on the 30 Fama–French industry portfolios from January 1996 to May 2007. The first half of the sample period is the optimization period and the second half is the out-of-sample period.

assets in their portfolios. For large portfolios ($n \geq 30$), however, the M-V portfolio performs better than the naïve portfolio. Exhibit 2 illustrates these results when $n = 15$. In Exhibit 2, the naïve portfolio is located above the M-V portfolio, but both are below the ex ante (i.e., unknown in advance) efficient frontier. Similar results (not shown here) for $n = 30$ reveal that the naïve and M-V portfolios almost coincide, with a small advantage to the M-V portfolio.

CONCLUDING REMARKS

Given the ex ante parameters, the Markowitz M-V rule, by definition, always dominates the naïve $1/n$ rule, which is the Talmudic strategy. In practice, however, ex ante parameters are generally estimated using historical data; therefore, it is not obvious that the naïve portfolio is inferior, because it has the advantage of not being exposed to the sampling errors of the various parameters. We find that the Talmudic wise men were correct for individual investors who hold relatively small numbers of assets in their portfolios. For institutional investors (large portfolios), the Markowitz M-V portfolio dominates the Talmudic portfolio. Interestingly, Markowitz, the "father" of modern portfolio theory, indirectly agreed with the Talmudic assertion by admitting that he uses the $1/n$ rule, justifying his contention as follows: "My intention was to minimize my future regret. So I split my contribution fifty-fifty between bonds and equities" (Zweig [1988]).

ENDNOTES

This article is adapted from a chapter of the *Handbook of Portfolio Construction,* edited by John Guerard, Jr., New York: Springer, 2009. For an extended version of this article, see Levy and Duchin [2003].

[1]The Babylonian Talmud, also called the Talmud Bavli, was transmitted orally for centuries prior to its compilation by Jewish scholars in Babylon about the fifth century.

[2]See also Levy and Sarnat [1972], p. 3.

[3]Levy and Markowitz [1979] showed that if returns fall in the range (–30%, +60%), the M-V rule (or quadratic preferences) provides an excellent approximation of the most commonly employed risk-aversion preferences (see also Markowitz [1991]). As in our study where most rates of return fall within this range, we can safely use the M-V rule.

[4]For details, see: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[5]Whenever $n < 30$, we consider the first n assets of the 30 Fama–French industry portfolios.

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